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# Technical Notes

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## Reynolds Analogy in High-Enthalpy and High-Mach-Number Turbulent Flows

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### Introduction

THIS Note presents the results of a shock-tunnel study in which skin friction, heat transfer rates, and static pressure are measured in hypervelocity turbulent boundary layers. The experiments are conducted at Mach numbers ranging from 4.2 to 5.9, flow stagnation enthalpies of 4.8–9.5 MJ/kg, and static pressures of 14–86 kPa. The objective of the present investigation is to explore the Reynolds analogy at high stagnation enthalpy and high Mach number over a broad range of $c_f$ values. Two previous shock-tunnel investigations[1,2] indicate that there is a decreasing measured Reynolds analogy factor with increased measured skin-friction coefficient at Mach numbers ranging from 4.6 to 7.1. The present Note presents results for Reynolds analogy factors at higher skin-friction coefficients than the previous studies and also includes results with nitrogen as a test gas to suppress real-gas effects.

### Experiment

#### Facility and Test Conditions

The experiments are carried out in the T4 free-piston reflected shock tunnel located at the University of Queensland. The facility has a 229-mm-diam driver tube that is 26 m long and a 75-mm-diam shock tube that is 10 m long. For the study, two contoured axisymmetric nozzles are used. The Mach 4 nozzle has an exit diameter of 135 mm and the Mach 6 nozzle has an exit diameter of 262 mm. After expansion through the nozzle, the test flow passes directly into the test section.

Eight test conditions are used to examine the Reynolds analogy in high stagnation enthalpy and high-Mach-number turbulent boundary layers. The stagnation enthalpy is determined from the incident shock speed and the initial shock tube filling pressure and temperature. If required, the assumption of an isentropic expansion of the air in the reflected shock region to the recorded mean nozzle supply pressure during the test time is also employed. A one-dimensional nonequilibrium nozzle expansion to the static pressure measured in the test duct yields the flow conditions in the test section.[3] All mainstream flow conditions at the first measurement location have a Reynolds number, based on the distance from the leading edge, that is greater than $2.0 \times 10^6$. Thus, based on evidence from previous experiments in the tunnel,[4,5] all boundary layers are assumed to be turbulent at the first measurement station for the present tests. However, for some tests, a boundary-layer trip as described by Mee[5] is attached 100 mm downstream of the leading edge of the inlet to ensure that the boundary layer is turbulent.

#### Test Section

Measurements of skin-friction coefficient and Stanton number are made within a 1745-mm-long duct that is shown in Fig. 1a. The test surface comprises one wall of the duct. The initial tests are conducted with an entry section that has the dimensions of 60 × 100 mm and a length of 245 mm. For the second phase of the investigation, the dimension of the entry section are 57 × 100 mm, as shown in Fig. 1b. The entry section is terminated by a 3-mm-high rearward-facing step downstream of which is a 1500-mm-long instrumented flat plate. The sidewalls of the test section diverge at 0.5 deg to account for the boundary-layer growth. For the Mach 6 flow, the width at the entry of the test section is increased from 100 to 120 mm to accommodate the increased size of the nozzle core flow. The divergence of the sidewalls is unchanged. The centerline of the instrumented flat plate is equipped with skin-friction gauges spaced at intervals of 100 mm. Thin-film heat transfer gauges and pressure transducers are located at intervals of 50 mm along lines parallel to the centerline. The heat transfer gauges are along a line that is 23 mm to the left of the centerline and the pressure transducers along a line that is 25 mm to the right.

#### Instrumentation

Surface shear stress measurements are made with skin-friction gauges that have a 10-mm-diam sensing disk that is mounted flush to the test surface. These gauges are designed and manufactured in-house and described in detail elsewhere.[6–9] The gauges incorporate an acceleration-compensated element and are individually calibrated to account for shear and pressure sensitivity. The heat transfer gauges are fabricated from a platinum thin film that is painted onto a quartz substrate and are also manufactured in-house. PCB™ piezoelectric pressure transducers are used to measure the surface static pressure.

The static pressure is measured adjacent to each skin-friction gauge, and this measurement is used to compensate the skin-friction gauge signal for pressure effects.

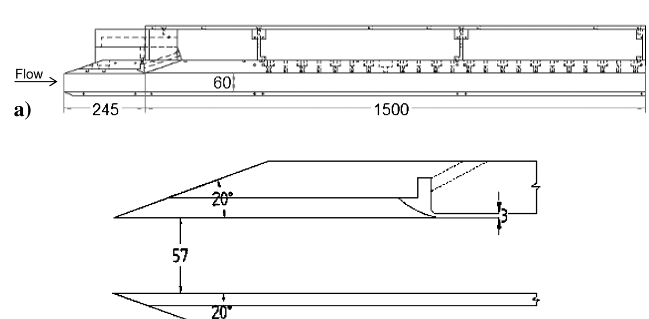


**Fig. 1 View of a) test section and b) entry section (dimensions in millimeters).**

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**Data Recording**

A 12-bit transient digital data acquisition and storage unit with a sampling time of 1 $\mu$s is used to acquire data. The outputs of the skin friction, heat transfer, and pressure transducers are multiplexed, resulting in a sampling time of 4 $\mu$s per channel. The thin-film gauge signals are reduced to heat transfer rates using the method of Schultz and Jones.[10]

## Experimental Data Analysis

For some conditions, a boundary-layer trip is located 100 mm downstream of the duct's leading edge. Measurements are also obtained without a boundary-layer trip for one test condition and show that within the experimental error there is no difference compared to the measurements with a boundary-layer trip.

Although attempts are made to minimize the effects of waves in the duct, there are variations of up to 20% in pressure along the test surface due to waves resulting from the viscous interaction, the 3-mm step in the duct, and the nonuniformities of the nozzle exit flows.

The local surface pressure is used to calculate local values of the flow speed and flow density outside the boundary layer; these are subsequently used to determine the local skin-friction coefficients and Stanton numbers. These local properties are calculated under the assumption that the flow is isentropically compressed or expanded to the measured pressure from its condition at entry to the duct.

Thus, the local skin-friction coefficient is given by

$$c_f = 2\tau_w \big/ \rho U^2 \tag{1}$$

The local heat transfer coefficient or Stanton number is given by

$$c_h = \dot{q} / \rho U (h_{aw} - h_w) \tag{2}$$

where $h_{aw} = h_0 + (r - 1)(U^2/2)$, $h_0$ is stagnation enthalpy, and $r = Pr^{1/3}$.

## Results

Results of measured Reynolds analogy factor with measured skin-friction coefficient are presented in Fig. 2. A trend of a decreasing Reynolds analogy factor with increasing skin-friction coefficient is apparent. The trend is independent of stagnation enthalpy, unit Reynolds number, and Mach number for the range of conditions that have been examined. This trend is not observed in flows with lower stagnation enthalpies. In fact, the Reynolds analogy factor increases with skin-friction coefficient when von Kármán's extended expression for Reynolds analogy factor in compressible flow is used.[11] The shock-tunnel experiments of Hironimus[1] also show a trend of decreasing Reynolds analogy factor as the skin friction is increased, although the maximum $c_f$ is less than $2 \times 10^{-3}$ and the range of $c_f$ values is relatively small. The results at the highest stagnation enthalpies, 1.30–3.47 MJ/kg, from Ref. 1, are shown in Fig. 2. Goyne et al.[2] determined, over a lower range of skin-friction coefficient, the Reynolds analogy factor in the same facility as the present study. The Mach 6 duct results from Ref. 2 are also shown in Fig. 2. The results of the present study and Ref. 2 are in good agreement, within experimental uncertainty, up to skin-friction coefficients of $2 \times 10^{-3}$. For skin-friction coefficients in the range $2 \times 10^{-3}$ and $3 \times 10^{-3}$, Reynolds analogy factors from the present study are approximately 15–30% lower than in the previous study.[2] However, there are approximately four times more measurements over this range of skin-friction coefficient in the present study compared to the previous study.[2] The trend of decreasing Reynolds analogy factor is not seen for $c_f$ values greater than $3 \times 10^{-3}$ because the measured levels are approximately constant.

The reason for the lower Reynolds analogy factors at high $c_f$ levels is unclear. The results indicate that surface shear stress levels are reasonably well predicted by the theory of Spalding and Chi[12] for all conditions, but the heat transfer rates are lower than predicted at conditions where $c_f$ is high. Some oxygen dissociation in the boundary layers would be expected at the conditions of the present experiment, and this may affect the heat transfer and skin-friction levels. Therefore, some tests are also conducted with nitrogen as the test gas to suppress real-gas effects. Results from those tests are also shown in Fig. 2 (open symbols) and are in close agreement with the results with air as the test gas.

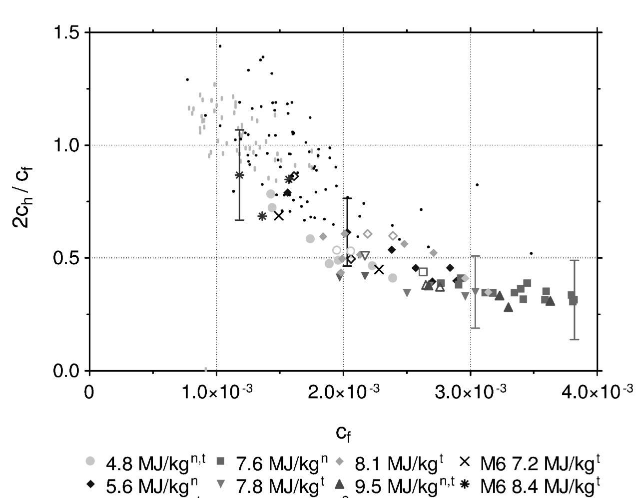


**Fig. 2 Effect of measured skin-friction coefficient on Reynolds analogy factor, $M = 4.2$–$5.9$, $Re_u = 3.7 \times 10^6$–$1.7 \times 10^7$ m$^{-1}$; superscript *n* denotes no trip, superscript *t* denotes trip, and M6 denotes Mach 6 conditions. Results with nitrogen as test gas, open symbols and results with air as test gas, filled symbols; representative error bars are shown.**

## Conclusions

Shock-tunnel measurements of skin-friction and heat transfer rates show a trend of decreasing Reynolds analogy factor with increasing skin-friction coefficient. This trend is apparently independent of stagnation enthalpy, unit Reynolds number, and Mach number for the range of conditions examined here. The results compare well from previous studies. However, at $c_f$ values between $2.0 \times 10^{-3}$ and $3.0 \times 10^{-3}$, the measured Reynolds analogy factor is consistently 15–30% below previous measurements. Measured Reynolds analogy factors for $c_f$ values greater than $3 \times 10^{-3}$ are approximately constant. The effect of oxygen dissociation within the boundary layer because as the source of the observed trend is discounted as similar Reynolds analogy factors are obtained when either air or nitrogen is used as the test gas. Further investigations are needed to clarify the mechanisms that produce these Reynolds analogy results.

## Acknowledgment

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